comments welcome, draft 11/20/17

Relational Possibility

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Absolute possibilities are about how things are at individual worlds. We might say that Socrates could have been a farmer or that Athens could have defeated Sparta. Relational possibilities are about how things compare or otherwise relate across worlds. We might say that Socrates could have been taller than he is or, perhaps, the Athenians could have been happier than they are.

Relational possibilities, while ordinary and familiar, turn out to be surprisingly hard to express systematically. The usual solution is to express them by quantifying over things like heights or degrees of happiness. As it turns out though, this natural solution has certain unexpected implications. One of them is that science is committed to far more ontology—to a wider variety of things—than we might otherwise have thought. The basic physical world will have to include not just particles with relations between them, but things like numbers, distances, or spacetime points.

I think we can do better. My aim in this paper is to convince you that we can understand relational possibilities without quantifying over things like heights. Not only does this better reflect our ordinary thinking about possibility, it also gives us a powerful strategy for doing science with minimal ontology.

1.

We have two ways of talking about possibility. The first is by using quantification over merely possible worlds or merely possible individuals. We might write $\exists w F(sw)$ to say that there is a possible world in which Socrates is a farmer or $\exists x(Cxs \wedge Fx)$ to say that there is a possible counterpart of Socrates who is a farmer. The second is by using modal operators instead. In that case, we would say that it could have been that Socrates was a farmer by writing $\Diamond F(s)$.

Corresponding to these two ways of talking, there are competing ideas about the nature of possibility. One is that possibility is ultimately about certain *things*—either possible worlds or possible individuals. The other is that possibility is ultimately about a certain *mode* or *manner* in which conditions are satisfied. The first view says that Socrates could have been a farmer because there is a possible world at which he is a farmer. The second says that no, on the contrary, the direction of explanation is reversed. There is a possible world at which Socrates is a farmer only because Socrates could have been a farmer.

Questions about relational possibility look somewhat different depending on whether we take worlds or modal operators to be more basic. For now, we are going to consider the matter from the perspective on which operators are most basic.

Suppose we have a language with a taller-than predicate, names for individuals, and a possibility operator. This lets us say that Aristotle is taller than Socrates by writing $T(as)$. We can then say that Aristotle could have been taller than Socrates by writing $\diamond T(as)$. Now we want to express the relational possibility of Socrates having been taller than he actually is. How should we do that? We could try $\diamond T(ss)$. But this says that Socrates could have been taller than himself, not that Socrates could have been taller than he is. We could try adding an actuality operator to our language. This might seem promising, since it lets us talk about the actual world within the scope of a possibility operator. We can then write $\diamond\downarrow T(ss)$. But this is true only if Socrates is *actually* taller than himself— which of course he is not. So it would seem that we have simply run out of syntactic combination. There is no way to say that Socrates could have been taller than he actually is. Call this the **problem of expression**.

One idea for solving the problem can be traced to Bertrand Russell's paper "On Denoting" (1905). Russell is there interested in understanding certain belief attributions. You might, on seeing a friend's new yacht, report that you had thought it would be larger than it is. Your friend is a bit touchy and so says no, obviously not. The yacht is *exactly* as long as it is. The joke gets its punchline because the target sentence is ambiguous between two readings. Russell thinks that what you intended to say is that the size you thought the yacht was is greater than the size the yacht is. Or in other words:

> There is a unique size y such that the yacht has y and you believed that (there is a unique size x the yacht has and $x > y$). (1)

The reading your friend attributes to you is the one on which the belief operator takes wide scope. That reading is of course absurd—no sane person would believe that the size of the yacht is greater than the size of the yacht.

The same strategy can be used to express relational possibilities. Suppose we replace belief with possibility and sizes with heights and the yacht with Socrates. We then have:

$$\text{There is a unique height } y \text{ Socrates has and } \Diamond(\text{there was a unique height } x \text{ Socrates had such that } x > y). \quad (2)$$

This would seem to be a perfectly good way of saying that Socrates could have been taller than he actually is. If we let the possibility operator take wide scope, we get the false claim that Socrates could have been taller than himself.

This solution obviously generalizes beyond sizes and heights. As we proceed, then, let's refer to things like sizes and heights as **degrees** and to Russell's strategy as the **degree solution**.

While the degree solution may solve the problem of expression, it also comes with certain metaphysical requirements. One of those requirements is that things like heights exist—we need *there to be* a height that Socrates has such that he could have had a greater one. The mere existence of heights, though, is not enough. Heights must also have a certain character.

When it comes to a quantity like height, we can distinguish two views. Absolutism says that height is ultimately about the determinate heights things have. Socrates has a height of six feet and this, the absolutist says, is among the most basic facts about height. Comparativists say no, on the contrary, height is ultimately about certain comparative relations like *being taller* and *being the same height*. The basic facts about height describe a web of such relations between individuals.[1]

The degree solution requires not just the existence of height, but the truth of absolutism. Here is why. Some comparativists deny flat-out that there are heights—our talk about Socrates having a height of six feet is just a confused remnant of discredited common sense. More likely, though, a comparativist will want to explain heights using height comparisons. For example, she might say that the height of Aristotle is identical to the height of Socrates because Aristotle is the same height as Socrates. Or she might say that the height of Socrates is six feet because he is the same height as certain standard reference objects—standard six foot measuring rods or something similar.

[1]See Dasgupta (2014) for further discussion.

Now a comparativist needs to explain de re modal facts about heights. Why could Aristotle have had the very height that Socrates actually has? The most straightforward answer is that the identity of heights across worlds works the same as the identity of heights across individuals. Aristotle could have had a height identical to the height Socrates actually has because Aristotle could have been as tall as Socrates actually is. But this means that we are explaining de re modal facts about heights using relational possibilities. Since the degree solution requires the reverse, it is no longer available. So the degree solution requires absolutism.[2]

Putting the matter more generally, the degree solution requires degrees to not just exist, but to be *prior* to relational possibilities. One implication is that the solution is not available to comparativists. Another, which we will consider in the next section, is that the solution prevents us from doing science with minimal ontology.

2.

Our best scientific theories describe a world built out of certain things, like quarks and bosons, that have certain features. We thus naturally think of science as having ontological commitments. Insofar as we believe our best theories, we are committed to believing in the things they describe—we are committed to believing in quarks and bosons and such. The question then is, how far do those commitments extend? What are the ontological requirements of science?

Science as we know it tells us about various physical quantities like mass, charge, and distance. Those quantities are described using numbers. To fix on an example, suppose we perform a series of experiments and discover that the movement of particles is described by Newton's laws of motion. These laws require distance ratios between particles to determine how they move.[3] What are distance ratios? Suppose we use a meter stick

[2]You could imagine a view on which the height of Aristotle at w is identical to the height of Socrates at v because Aristotle occupies the same position in the network of height relations at w as Socrates does at v. We would then be free to use heights to explain relational possibilities, since they are not presupposed. In that case, though, the appeal to heights is redundant. We can just say that Aristotle at w is the same height as Socrates at v because he occupies the same position in the network of height relations. So more carefully, we might say that the degree solution is *useful* only if absolutism is true.

[3]Here is a simple case. Suppose the world is Newtonian with gravity the only force. There are three particles abc with b between ac. The particles are at rest relative to one another and a is as massive as bc put together. Then, a and c will collide simultaneously with b just in case the distance ratio of ab to bc is $\sqrt{2}$. If there is no determinate distance ratio, the laws fail to determine whether the particles will collide simultaneously.

to determine that *ab* are two meters apart and *bc* are one meter apart. We could then record the result using a distance ratio function from particles to numbers by writing $\delta(abbc) = 2$. If you like, think of this as a certain definite description:

$$\text{The distance ratio of } ab \text{ to } bc = 2. \tag{3}$$

Such measurements let us apply the laws and predict how things move.

How should we understand such distance ratios? One view is that we should take science at face value. Distance ratios ultimately involve a relation between particles and numbers and, so, numbers play an essential role in the physical world. Science is as much committed to numbers as it is to quarks and bosons. A competing view is that numbers are useful but not essential. They are part of science only because they speed along certain reasoning. With enough time and patience, we could fully describe the physical world without them. There may be physical facts involving numbers like (3). But if so, they will have to be ultimately explained using physical facts not involving numbers.[4,5]

Suppose we agree that numbers are not essential to the physical world. We then face the challenge of showing how there can be distance ratios without numbers. One strategy for doing that requires nothing more than particles and a pair of spatial relations. Those relations are betweenness and congruence. Intuitively speaking, betweenness is the relation of one thing being on a straight line between two others. Distance congruence is the relation of two things being exactly as far apart as two others. Now imagine a world with exactly four point particles that looks like this:

(4)

The betweenness and congruence relations of the world are as they appear. The particle *x* is between *ab*, the particles *ax* are congruent with *xb*, and so on. It also looks like the ratio of the distance between particles *ab* and

[4]The first view is endorsed by Quine (1960, 1963). Harty Field (1980) defends the second.

[5]As Field (1984) points out, there are different sorts of reasons you might think that numbers are dispensable. You might be a nominalist who denies flat-out that numbers exist. Alternatively, you might accept the existence of numbers, but deny that there are any basic physical relations between physical things and numbers. Field calls this latter view moderate platonism. Nominalists, then, need to show why scientific claims involving numbers are useful without being true. Moderate platonists need to show why they are true using only physical facts that do not involve numbers.

bc is two, and in fact they are. What we want to do is explain this distance ratio in terms of the basic congruence and betweenness relations.

Just looking at the illustration, you can essentially see how the explanation goes. Each adjacent pair of particles is congruent with any other adjacent pair, so we can treat those pairs as "units". There are then two "units" between *ab*, but only one "unit" between *bc*. So *ab* are twice as far apart as *bc*. A bit more carefully, say that x is halfway between *ab* when x is between *ab* and *ax* and *xb* are congruent. We then claim that *ab* are twice as far apart as *bc* because:

> There is an x halfway between *ab* such that *ax* and *bc* are congruent. (5)

Other rational distance ratios can be defined similarly. Irrational distance ratios are defined using limits.

All of this would seem to work beautifully. The problem is that the proposed reduction works only if there happen to be enough particles and they happen to be in the right place. Suppose we have a world just like the last except that we delete the second particle.

(6)

This is a world in which *ab* is twice as far apart as *bc*. Our proposed definition of that distance ratio, though, requires there to be something halfway between *ab*. Since there is nothing there, the definition fails.[6]

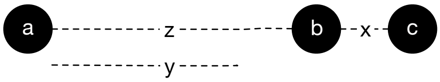
The problem can be fixed if we accept the existence of spacetime. Spacetime points, like particles, stand in betweenness and congruence relations. Unlike particles, though, spacetime points are highly organized—you can always count on them to be where they need to be. In particular, there will be laws guaranteeing that whenever there are two things, there is a spacetime point halfway between them. Adding spacetime points to our three particles world, then, we have a world with a spacetime point x halfway between *ab*.

(7)

[6]You might wonder whether this is just a failure of imagination. Sure, one proposed reduction to betweenness and congruence fails, but maybe there is some other reduction? In fact there is not. Consider a three particle world just like the one illustrated except that *ab* is half as far apart as *bc* instead of twice as far apart. That world has exactly the same betweenness and congruence relations but has different distance ratios. So, there is no way to explain the difference in distance ratios between those worlds using only betweenness and congruence.

Because there is once again *something* halfway between *ab*, our proposed definition of the distance ratio now works just fine.[7]

A different kind of strategy is to accept the existence of distances rather than spacetime points. What are distances? There are different views, but the simplest is that distances are first-order universals. We then have two important relations. The first is a three-place addition relation between distances. This is just the relation of distances *xy* together being as long as *z*. The second is a having relation between pairs of particles and distances. To see how this all works, consider a world in which there are three particles, with *ab* three times as far apart as *bc*.

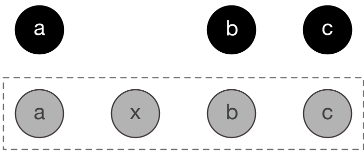


(8)

Here, particles *ab* have distance *z* and *bc* have distance *x*. The distance *y* is one that no particles have. We can then easily explain why *ab* is three times as far apart as *bc*. That is because *x* and *x* add up to *y*, and *x* and *y* add up to *z*, with *ab* having *z* and *bc* having *x*. Note especially the importance of distances that no particles have. Without distance *y*, there would be no way to make this work.[8]

We can explain distance ratios without numbers, then, using either spacetime or distances. In either case, science is ontologically committed to more than just particles. You might wonder, then, whether these are the only options. Can we explain distance ratios using only particles? Or does science inevitably require further things?

The most obvious strategy to try is trading ontology for possibility. Consider the following world for example. The black dots represent actual particles and the box with grey dots represents a certain possibility.



(9)

[7]Early attempts to axiomatize Euclidean space using betweenness and congruence include (Veblen, 1904) and (Pieri, 1908). The project was later advanced by Alfred Tarski and his students, who give increasingly simple axiom schemes in (Tarski, 1952), (Tarski, 1959), and (Gupta, 1965). I owe the basic argument that distance ratios require either numbers or spacetime points to Field (1984).

[8]Brent Mundy (1987) describes a variation. For him, distances are binary relations instead of first-order predicables. He then has second-order quantification over these relations and a second-order addition relation between them. Myself, I prefer the first-order version. I can see no advantage that would justify the added cost of second-order ideology.

The idea is that particles *ab* are twice as far apart as *bc*, not because there is actually something halfway between *ab* meeting the right conditions, but because there *could* have been.

This modal strategy, though, would seem to be doomed. We are trying to avoid further ontology, so the relevant possibility has to be expressed using operators rather than worlds. As a first try:

> It could have been that there was an *x* halfway between *ab* such that *ax* were as far apart as *bc*. (10)

But this is completely trivial. Even if *ab* are a thousand times as far apart as *bc*, *a* could have been closer to *b*. And in that case, there also could have been an *x* halfway between *ab* such that *ax* were as far apart as *bc*. What we need is a possibility that "holds fixed" the actual particles. For that, we need a certain relational possibility.

> It could have been that *ab* were as far apart as they actually are and that *bc* were as far apart as they actually are and that there was an *x* halfway between *ab* such that *ax* was as far apart as *bc*. (11)

The degree solution is the only solution we have for expressing relational possibilities with operators. Expressing such possibilities, then, requires quantifying over things that can play the role of degrees. Which things? We could use distances, numbers, or chunks of spacetime. But then we are just back where we started! We are quantifying over precisely the things whose existence we want to avoid. Science, it would seem, inevitably requires more than just particles.

The right response, I think, is to find a better solution to the problem of expression. If we can express relational possibilities without degrees, we can give a theory of distance ratios without numbers, spacetime, or distances. The result is a powerful general strategy for doing science with minimal ontology. The rest of this paper will be spent developing such a solution.

3.

Start with standard first-order language. It has quantifiers and predicates and names for individuals and so on. We then add a possibility operator $\Diamond$ and define the necessity operator $\Box = \neg\Diamond\neg$. There is no way to express relational possibilities directly, though perhaps we can express them indirectly if we quantify over degrees. That of course is the whole problem. Call this language **absolute modalese**.

One of the best reasons to think that we can express relational possibilities with operators and without degrees is that we can do it in English. Consider first the claim that:

It could have been that Aristotle was taller than Socrates was. (12)

This sentence expresses an absolute possibility. It is built out of a sentential possibility operator, a two-place predicate, and a pair of names. The key feature is that the predicate has two copulas, both of which are in the subjunctive mood. We say that it could have been that Aristotle *was* taller than Socrates *was*.[9] There is no need for quantification over either worlds or degrees. If we put the second copula in the indicative mood instead, we get:

It could have been that Aristotle was taller than Socrates is. (13)

Now we have a sentence expressing a relational possibility—all we had to do was change the mood of the second copula.[10] English, then, can express relational possibilities using nothing more than modal operators, predicates, names, and grammatical mood.[11]

This suggests a strategy for solving the problem of expression. Absolute modalese has modal operators, predicates, and names, but has nothing playing the role of grammatical mood. If we can find something to play that role, we should be able to express relational possibilities.

Even setting aside comparisons, there are signs that absolute modalese has a grammatical mood problem. We can say in English that:

It could have been that everyone who isn't a farmer was a farmer. (14)

If both copulas in the scope of the operator were in the subjunctive mood, the sentence would be inconsistent—it could not have been that everyone who *wasn't* a farmer *was* a farmer. Putting the first copula in the indicative

[9]The subjunctive mood in English is expressed using what is sometimes called *fake past tense*. You can see that phenomenon here. We conjugate 'is' as 'was' but, rather than saying something about the actual past, we are saying something about the non-actual present. See Iatridou (2000) for further discussion. One of the hazards of this system of expression is that sentences often have multiple readings depending on whether we read the verb conjugation as fake past tense or real past tense. For our purposes, we can just stipulate that the intended reading is always fake past tense.

[10]We can emphasize the intended reading by saying that it could have been that Aristotle was taller than Socrates *actually* is. But semantically speaking, this is not required.

[11]See Lewis (1986) pp. 13-4, Wehmeier (2003), and Mackay (2013) for further discussion.

mood results in a sentence that is not just consistent, but true. In terms of worlds, it says that there is a possible world w such that every x in the actual world v who is not a farmer at v is a farmer at w. There is no way to say that in absolute modalese, though, and adding an actuality operator is no help.[12]

4.

Set aside modal operators for a moment. Suppose we wanted to express possibilities with worlds instead. How would we do that? What language would we use?

One option is to use **absolute possiblese**. This is a two-sorted language that has a set of variables $\{x, y, ...\}$ ranging over individuals and another $\{w, v, ...\}$ ranging over worlds. Names are similarly sorted. This sorting of terms is important because it helps determine which terms can go where. Predicates have sorted argument places, so each argument place can take either individual terms or world terms, but not both. Besides identity, there are three logical predicates $A(w)$, $I(xw)$, and $R(wv)$. The first says that a world w is actual. The second says that individual x is in a world w. The third says that a world w is possible relative to v.

The key feature of absolute possiblese is that all non-logical predicates are indexed to worlds. This means that they all have the form $P(x_1, ..., x_n, w)$. So for example, where we might ordinarily have a predicate $F(x)$ saying that x is a farmer, absolute possiblese has a predicate $F(xw)$ saying that x is a farmer at w.[13]

Using absolute possiblese is incredibly straightforward. We can say that Socrates is not a farmer at the actual world. We can say that there is a possible world at which Socrates is a farmer. And we can say that there is a possible world at which all the Athenians are happy. These sentences are all essentially just direct transcriptions of what we say in English.[14]

[12] $\Diamond\forall x(\neg\downarrow Fx \supset Fx)$ is tempting, but not what we want. Suppose Socrates is the only person who is not a farmer. Suppose the only merely possible world is one in which Socrates does not exist. The claim using the actuality operator is then true, but (14) is false, at least on its most natural reading. The claim in English is true only if there is a possible world in which all of the actual non-farmers exist and are farmers. See Hazen (1976) for further discussion of the expressive limitations of absolute modalese.

[13] Absolute possiblese is just a standard two-sorted language, so has standard two-sorted models. The only additional requirement is that those models have a unique actual world. See Gallier (2015) for an introduction to many-sorted logic.

[14] $\exists v(Av \wedge Fsv)$ and $\exists wv(Av \wedge Rwv \wedge Fsw)$ and $\exists wv[Av \wedge Rwv \wedge \forall x(Ixw \wedge Ax \supset Hx)]$ respectively.

You can probably already see that absolute possiblese has a problem of expression. The taller-than predicate, like other non-logical predicates, will be indexed to a single world. So it has the form $T(xyw)$. We can then say that Aristotle is taller than Socrates at the actual world. We can say that there is a possible world at which Aristotle is taller than Socrates.[15] What we cannot say is that there is a possible world at which Aristotle is taller than Socrates is at the actual world. There is no way to directly express even simple relational possibilities.

This is not surprising. As it turns out, there is a natural correspondence between absolute modalese and absolute possiblese. We all know how to translate claims involving modal operators into claims involving worlds, and visa versa. If you say that it could have been that Socrates was a farmer, I know that the corresponding claim is that there is a possible world in which Socrates is a farmer. If I say that there is a possible world in which all the Athenians are happy, you know that the corresponding claim is that it could have been that all the Athenians were happy.

These sorts of translations can be regimented using a kind of standard translation manual. That translation manual includes a recursive function $St(\cdot)$ translating every sentence of absolute modalese into a sentence of absolute possibles. And it includes another recursive function $Tr(\cdot)$ that translates a well-defined fragment of absolute possiblese back into absolute modalese.[16] We might then think of absolute possiblese and absolute modalese as being linguistic counterparts. One uses modal operators and the other uses worlds, but they express the same sort of possibilities.[17]

Now in the case of absolute possiblese, the problem of expression is easy to solve. Rather than indexing non-logical predicates to a single world, we simply index them to a pair of worlds. Our taller-than predicate then has the form $T(xywv)$ instead of $T(xyw)$. This lets us say that there is a possible world at which Socrates is taller than he is at the actual world.[18] No problem at all. Call this revised language **relational possiblese**.

5.

Now back to operators. Start again with a first-order language, adding a possibility operator and defining the necessity operator as before. This

[15] These two claims are $\exists v(Av \wedge Tasv)$ and $\exists wv(Av \wedge Rwv \wedge Tasv)$.

[16] These translation functions are described in the second appendix.

[17] In modal logic, the study of such translations is called correspondence theory. The first-order language is then sometimes called the correspondence language. See Blackburn and van Benthem (1988).

[18] $\exists wv(Av \wedge Rwv \wedge Tsswv)$

time, rather than having just modal operators, we will also have **mood operators**. Those are $\otimes$ and $\downarrow$ with a third $\uparrow = \otimes\downarrow$ defined in terms of them. Call these operators swap, down, and up respectively. The resulting language is **relational modalese**.

As it turns out, relational modalese and relational possiblese are linguistic counterparts. The two languages match in exactly the way that absolute modalese and absolute possiblese match. Think of adding mood operators, then, as the operational equivalent of adding a second world index to predicates.

We have a new language, then, but how does it work? Here is the intuitive idea. Imagine that worlds are planets and that you are standing at the actual world next to a giant telescope. Sentences of absolute modalese can then be thought of as ascribing properties to worlds. When you assert that P, you are saying that your world has property P. The diamond operator then lets you describe what you can see through the telescope. When you say that $\Diamond P$, you are saying that you can see a world that has property P.

Relational possiblese is a little different. Rather than ascribing properties to worlds, it ascribes binary relations to worlds. When you say that R, you are saying that your world stands in relation R to itself. When you say that $\Diamond(R)$, you are saying that you can see a world through the telescope that stands in relation R to your own world.

Mood operators expand the range of features you can describe. Swap lets you "flip" the relation around. Saying $\Diamond(\otimes R)$ means that you can see a world with *your* world standing in relation R to *that* world. Up is then used to describe the intrinsic features of other worlds. Saying that $\Diamond(\uparrow R)$ means that you can see a world that stands in R to itself. Down is used to describe the intrinsic features of the actual world. Saying that $\Diamond(\downarrow R)$ just means that you can see a world such that (ignoring that world) your own world stands in R to itself.

Our mood operators are aptly named, I think, because this is more or less the role grammatical mood plays in English. When we say that it could have been that Aristotle *was* taller than Socrates *was*, the sentence inside the scope of the possibility operator is uniformly in the subjunctive mood. This indicates that we are expressing a pure possibility, one not involving the actual world. This is what the up operator does in relational possiblese. We say that Aristotle could have been taller than Socrates was by writing $\Diamond\uparrow T(as)$.

When we say that it could have been that Aristotle *was* taller than Socrates *is*, the sentence inside the scope of the possibility operator is in mixed mood. This indicates that we are expressing an impure possibility, one involving a possible world and the actual world. This is what the possibility operator in relational possiblese does by default. To say that Socrates could have been taller than he is, then, we simply write $\Diamond T(as)$.

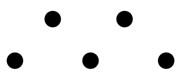
Notice that we get a kind of shift moving from absolute to relational modalese. In absolute modalese, we say that Aristotle could have been taller than Socrates by writing $\Diamond T(as)$. In relational modalese, that same claim requires the up operator. This is completely general. Think of the diamond in absolute modalese as being equivalent to $\Diamond\uparrow$ in relational modalese.

This solves the problem of expression. We can express relational possibilities and we can do it without degrees. As a bonus, we can even express non-comparative claims like (14). We can say that it could have been that everyone who isn't a farmer was a farmer.[19]

For those interested, the first appendix gives a formal semantics for relational modalese using Kripke models. The second appendix gives standard translations from relational modalese into relational possiblese, and visa-versa. This shows that those languages really do match in the same way that absolute modalese and absolute possiblese match. After the second appendix, there is a table associating sentences of modalese with their translations into possiblese. Myself, I have found that seeing these sentences together is a good way to get a feel for how relational modalese works and the sort of claims it can express.

6.

We have all been told, at one time or another, that possibility is about how things could have been. This is the view more or less built into absolute modalese. Possibilities are about how things could have been in themselves and can be fully described independently of how things are. When I think of possibility as ways things could have been, I imagine our world as one of many dots in modal space. Like this:

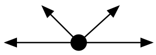


(15)

Relational modalese ultimately represents not just a different way of talking about possibility, but a different way of thinking about its nature. Possibilities are ultimately not ways things could have been, but ways

[19] $\Diamond\otimes\forall x\otimes(\neg\downarrow Fx \supset \uparrow Fx)$

things could have *differed*. When I think about possibility this way, I imagine vectors with our world as the origin.



(16)

Rather than being points that can be independently characterized, possibilities are departures or displacements from reality.

What we have seen is that given the first picture, relational possibilities come with ontological commitments. On the second, we can do without those commitments. The question then is, why did we find the first picture so compelling in the first place?

Let's try a pair of experiments. First, close your eyes. Imagine that Holmes and Dr. Watson are sitting in their favorite easy chairs at 221 Baker Street. Holmes is holding up his thumb and long, slightly crooked index finger. His fingers are about three inches apart and exactly halfway between them is a blue marble suspended in midair. "Isn't that curious," he tells Dr. Watson.

Now the second. Hold your thumb and index finger two or three inches apart and focus on the point halfway between them. Imagine there is a blue marble suspended there, spinning slowly and reflecting the light. Stare at that point for awhile and imagine as much detail as you can. Can you see it? I find that I can almost see a ghostly hologram.

These two experiments represent two ways of imagining—two modes of conceivability if you will. In the first case, we close our eyes and build a world from scratch. Maybe we can stipulate that we are imagining things about certain individuals. We can imagine that Humphrey himself wins the election, not just that a Humphrey lookalike wins the election. What we cannot do is directly compare what is imagine with what is actual. When we close our eyes to picture a world, the slate is wiped clean. All we have is empty imaginative space. The actual world has gone away.

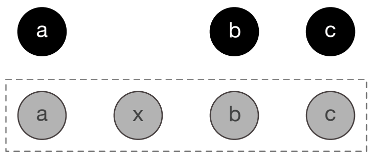
In the second case, we imagine a possible world while keeping our eyes open. Instead of imagining a possibility by building from scratch, we imagine a possibility "superimposed" on the actual world. This lets us imagine certain relational possibilities directly. When you imagine a marble halfway between your fingers, you are imaging that your fingers were as far apart as they actually are and that there was a marble halfway between them. This is is just a direct imaginative act, or so it seems to me. Your ability to imagine such things does not depend on numbers, spacetimes, distances, or the like.

We philosophers, I think, tend to imagine worlds in the first way. This tends to encourage the idea that possibility is about ways things could have been. But once you notice that we can imagine relational possibilities directly in the second way, it is just as natural to think of them as ways things could have been. We can think of them as contrasts with or perhaps displacements from reality.

7.

Now that we have a new way of thinking and talking about possibility, we can return to the question of distance ratios. We are going to show how, exactly, this helps us do science with minimal ontology.

In section two, you will remember, we saw that it was relatively easy to explain distance ratios using numbers, spacetime, or distances. What we would like is an explanation that does without such things. One strategy for doing that is to use modality. The intuitive idea is to imagine that the world looks something like this:



(17)

The black dots are the actual particles. Particles *ab* are twice as far apart as *bc*. The box with grey dots represents a certain possibility. This is the relational possibility of (a) *ab* having been as been far apart as they are and *bc* having been as far apart as they are with (b) a particle *x* having been between *ab* such that *ax* were as far apart as *xb* and *xb* were as far apart as *bc*. We then say that what it is for *ab* to be twice as far apart as *bc* is for this modal fact to obtain.

The problem we had before is that we had no way of expressing the needed possibility without using degrees. Now we do. We can use relational modalese to write:

$$\Diamond(Cabab \wedge Cbcbc \wedge \uparrow\exists x(Baxb \wedge Caxxb \wedge Caxbc)) \tag{18}$$

How does this sentence work? The predicates B and C are betweenness and congruence predicates. The first half $\Diamond(Cabab \wedge Cbcbc \wedge ...)$ then describes a possibility in which the actual particles are held fixed. It says that it could have been that *ab* were as far apart as they are and that *bc* were as far apart as they are and... That condition is in "mixed mood" because there are no mood operators. The second half $\Diamond(...\uparrow\exists x(Baxb \wedge$

$Caxxb \wedge Caxbc$)) describes how things are within the box. It says that it could have been that...there was a particle x that was between ab such that ax were as far apart as xb were and ax were as far apart as bc were. That condition is uniformly in the "subjunctive mood" because everything is inside the scope of the up operator. Putting these two halves together gives us the possibility we need.

We have then a kind of proof of concept. We can use relational possibilities to explain distance ratios without numbers, spacetime, or other such things. All we need are particles, a pair of relations between them, and relational modalese. This is of course not the end of the story. Science is more than distance ratios, and there is far more to say about the idea that possibilities are ways things could have differed. But we are well on our way. We have a promising strategy for doing science with minimal ontology.

Appendix: Semantics

Kripke structures $\langle W, R, D, d, @\rangle$ are familiar. They include a domain of worlds, a binary accessibility relation on those worlds, a domain of individuals, a domain function, and a designated actual world. In other words, W and D are non-empty and $R \subset W \times W$ and $d \colon W \to \mathcal{P}(D)$ and $@ \in W$.

A model of relational modalese is a Kripke structure together with an interpretation $[\![\cdot]\!]$. That interpretation assigns every name to an individual. It also assigns every predicate to a function $W \times W \mapsto \mathcal{P}(D^n)$ called a binary intension. We can then recursively define truth at a world w relative to a world v and a variable assignment σ. Like this:

$$wv \models_\sigma P(t_1, \ldots, t_n) \text{ iff } \langle \sigma(t_1, w), \ldots, \sigma(t_n, w)\rangle \in [\![P]\!](wv)$$
$$wv \models_\sigma \neg\phi \text{ iff } wv \not\models_\sigma \phi$$
$$wv \models_\sigma \phi \wedge \psi \text{ iff } wv \models_\sigma \phi \text{ and } wv \models_\sigma \psi$$
$$wv \models_\sigma \exists x\phi \text{ iff } wv \models_\tau \phi \text{ for some } \tau$$
$$wv \models_\sigma \Diamond\phi \text{ iff there is a } u \text{ with } R(uw) \text{ and } uw \models_\sigma \phi$$
$$wv \models_\sigma \otimes\phi \text{ iff } vw \models_\sigma \phi$$
$$wv \models_\sigma \downarrow\phi \text{ iff } vv \models_\sigma \phi$$
$$wv \models_\sigma \uparrow\phi \text{ iff } ww \models_\sigma \phi$$

Note that variable assignments are relative to worlds because different individuals are in different worlds. So $\sigma(t, w) \in d(w)$ when t is a variable and $\sigma(t, w) = [\![t]\!]$ when t is a name. We then say that ϕ is true in a model full stop when $@@ \models_\sigma \phi$ relative to all variable assignments σ.

Something worth emphasizing, I think, is just how similar this is to absolute modalese. An absolute modalese model is also a Kripke structure together with an appropriate interpretation. That interpretation assigns every name to an individual and every predicate to a monadic intension $W \mapsto \mathcal{P}(D^n)$. We then define truth at a world w relative to a variable assignment.

$$w \models_\sigma P(t_1, \ldots, t_n) \text{ iff } \langle \sigma(t_1, w), \ldots, \sigma(t_n, w)\rangle \in [\![P]\!](w)$$
$$w \models_\sigma \neg\phi \text{ iff } wv \not\models_\sigma \phi$$
$$w \models_\sigma \phi \wedge \psi \text{ iff } wv \models_\sigma \phi \text{ and } wv \models_\sigma \psi$$
$$w \models_\sigma \exists x\phi \text{ iff } wv \models_{\sigma*} \phi \text{ for some } \sigma*$$
$$w \models_\sigma \Diamond\phi \text{ iff there is a } u \text{ with } R(uw) \text{ and } uw \models_\sigma \phi$$

A sentence ϕ is true in a model when $@ \models_\sigma \phi$ for all variable assignments.

The relational character of relational modalese comes out in two ways. One is that predicates are assigned to binary intensions rather than monadic intensions. The other is that we use pairs of worlds rather than just worlds when giving truth conditions. But otherwise, the general style of semantics is just the same.

Appendix: Translations

In this appendix, we are going to describe the translation manuals for both absolute and relational modalese. You will remember that each translation manual includes two translation functions. One goes from modalese into possiblese. The the other goes back the other direction.

We are going to start with translations from modalese into possiblese. The standard translation for absolute modalese is defined using a pair of mutually recursive functions as specified below by (1),(2) and (4)-(7). The standard translation for relational modalese is given by (1) and (3)-(9).

(1)	$ST(\phi) = \exists v(Av \wedge [\phi]^1(w/v))$	
(2)	$[P(\bar{t})]^1 = P(\bar{t}v)$	$[P(\bar{t})]^2 = P(\bar{t}w)$
(3)	$[P(\bar{t})]^1 = P(\bar{t}vw)$	$[P(\bar{t})]^2 = P(\bar{t}wv)$
(4)	$[\phi \wedge \psi]^1 = [\phi]^1 \wedge [\psi]^1$	$[\phi \wedge \psi]^2 = [\phi]^2 \wedge [\psi]^2$
(5)	$[\neg\phi]^1 = \neg[\phi]^1$	$[\neg\phi]^2 = \neg[\phi]^1$
(6)	$[\exists x\phi]^1 = \exists x(Ixv \wedge [\phi]^1)$	$[\exists x\phi]^2 = \exists x(Ixw \wedge [\phi]^2)$
(7)	$[\Diamond\phi]^1 = \exists w(Rwv \wedge [\phi]^2)$	$[\Diamond\phi]^2 = \exists w(Rwv \wedge [\phi]^1)$
(8)	$[\downarrow \phi]^1 = [\phi]^1(v/w)$	$[\downarrow \phi]^2 = [\phi]^2(w/v)$
(9)	$[\otimes\phi]^1 = [\phi]^2$	$[\otimes\phi]^2 = [\phi]^1$

Note that $\bar{t}$ indicates an arbitrary sequence of terms and (w/v) indicates that free occurrences of w are uniformly replaced with v.[20]

Standard translations like those in Blackburn and van Benthem (1988) have the curious feature of translating closed sentences of modalese into *open* sentences of possiblese. Maybe such open sentences can be thought of as making indexical assertions. *This* world is one from which there is an accessible world at which Socrates is a farmer. What we would really like, though, are translations from non-indexical claims in modalese to non-indexical claims of possiblese. This way of doing standard translations does that. Blackburn and van Bentham also use infinitely many mutually recursive functions, since they have one for every world variable in the

[20]Here is an example of the standard translation for relational modalese in action: Start with $\Diamond\uparrow Tas$, the claim that Aristotle could have been taller than Socrates. That is equivalent to $\Diamond\downarrow \otimes Tas$ by the definition of $\uparrow$. We then have $ST[\Diamond\downarrow \otimes Tas] = \exists v(Av \wedge [\Diamond\downarrow \otimes Tas]^1) = \exists v(Av \wedge \exists w(Rwv \wedge [\downarrow \otimes Tas]^2) = \exists v(Av \wedge \exists w(Rwv \wedge [\otimes Tas]^2(v/w)) = \exists v(Av \wedge \exists w(Rwv \wedge [Tas]^1(v/w)) = \exists v(Av \wedge \exists w(Rwv \wedge (Tasvw)(v/w)) = \exists v(Av \wedge \exists w(Rwv \wedge (Tasww))$.

possiblist language. This is massive overkill, since only two are needed. Their translation is for standard propositional modal logic, so ours also differs by including clauses for predicates, quantifiers, and the new down and swap operators.

Not every sentence in possiblese has a translation into modalese. In possiblese we can say that there are exactly 28 impossible worlds in which George Harrison is an avocado—just try saying that in modalese. This means that we need to specify *which* sentences of possiblese are in the domain of the translation function $Tr(\cdot)$ going the other direction.

Here's how to do that. Suppose we start with the syntax for absolute possiblese, but use restricted formation rules instead of the standard formation rules.

> If ϕ is atomic and does not include $R(wv)$, $I(xw)$, $A(w)$, or identity for worlds, then ϕ is a wff.[21]
> If ϕ and ψ are wffs, then $\phi \wedge \psi$ is a wff.
> If ϕ is a wff, then $\neg\phi$ is a wff.
> If ϕ is a wff, then $\exists x(Ixw \wedge \phi)$ is a wff.
> If ϕ is a wff with w the only free variable, then $\exists w(Aw \wedge \phi)$ is a wff.
> (AP) If ϕ is a wff with w the only free variable and w, v distinct, then $\exists w(Rwv \wedge \phi)$ is a wff.

Call the resulting fragment of absolute possiblese **minimal AP**. In the case of relational possiblese, we do the same, but replace (AP) with (RP).

> (RP) If ϕ is a wff with w, v the only free variables and w, v distinct, then $\exists w(Rwv \wedge \phi)$ is a wff.

This gives us a fragment of relational possiblese we will call **minimal RP**.

Now we can give our second translation function $Tr(\cdot)$. In the case of absolute possiblese, that translation has minimal AP as its domain and, in the case of relational possiblese, it has minimal RP. Because open sentences never have more than two free variables in either language, every sentence is equivalent to one in which the only variables are w and v. Variable substitution is recursive, so we only need translation clauses for sentences in which these are no more than two variables.

[21] Identity for worlds means expressions of the form $t = s$ in which either t or s are terms for worlds.

(1)	$Tr(\phi) = [\phi]^1$	
(2)	$[P(tw)]^1 = P(t)$	$[P(tv)]^2 = P(t)$
(3)	$[P(twv)]^1 = P(t)$	$[P(tvw)]^2 = P(t)$
(4)	$[P(tvw)]^1 = \otimes P(t)$	$[P(twv)]^2 = \otimes P(t)$
(5)	$[P(tvv)]^1 = \downarrow P(t)$	$[P(tww)]^2 = \downarrow P(t)$
(6)	$[\phi \wedge \psi]^1 = [\phi]^1 \wedge [\psi]^1$	$[\phi \wedge \psi]^2 = [\phi]^2 \wedge [\psi]^2$
(7)	$[\neg\phi]^1 = \neg[\phi]^1$	$[\neg\phi]^2 = \neg[\phi]^2$
(8)	$[\exists v(Rvw \wedge \phi)]^1 = \Diamond[\phi]^2$	$[\exists w(Rwv \wedge \phi)]^2 = \Diamond[\phi]^1$
(9)	$[\exists w(Rwv \wedge \phi)]^1 = \otimes\Diamond\otimes[\phi]^2$	$[\exists v(Rvw \wedge \phi)]^2 = \otimes\Diamond\otimes[\phi]^1$
(10)	$[\exists x(Ixw \wedge \phi)]^1 = \exists x[\phi]^1$	$[\exists x(Ixv \wedge \phi)]^2 = \exists x[\phi]^2$
(11)	$[\exists x(Ixv \wedge \phi)]^1 = \otimes\exists x\otimes[\phi]^1$	$[\exists x(Ixw \wedge \phi)]^2 = \otimes\exists x\otimes[\phi]^2$
(12)	$[\exists v(Av \wedge \phi)]^1 = [\phi]^2$	$[\exists w(Aw \wedge \phi)]^2 = [\phi]^1$
(13)	$[\exists w(Aw \wedge \phi)]^1 = [\phi]^1$	$[\exists v(Av \wedge \phi)]^2 = [\phi]^2$

The translation function for relational possiblese is defined by (1) and (3)-(13). For absolute possiblese, the translation is the same, but uses (2) instead of (3) and does not need either (9) or (11).

English	Absolute Modalese	Absolute Possiblese
Aristotle is taller than Socrates.	$T(as)$	$\exists v(Av \wedge Tasv)$
Aristotle could have been taller than Socrates.	$\Diamond(Tas)$	$\exists v[Av \wedge \exists w(Rwv \wedge Tasw)]$
It could have been that every farmer was a philosopher.	$\Diamond\forall x(Fx \supset Px)$	$\exists v[Av \wedge \exists w(Rwv\wedge \forall x(Ixw \supset (Fxw \supset Pxw)))]$

English	Relational Modalese	Relational Possiblese
Aristotle is taller than Socrates.	$T(as)$	$\exists v(Av \wedge Tasvv)$
It could have been that Aristotle was taller than Socrates is.	$\Diamond(Tas)$	$\exists v[Av \wedge \exists w(Rwv \wedge Taswv)]$
It could have been that Aristotle was taller than Socrates was.	$\Diamond\uparrow(Tas)$	$\exists v[Av \wedge \exists w(Rwv \wedge Tasww)]$
It could have been that Socrates was shorter than Aristotle is.	$\Diamond\otimes(Tas)$	$\exists v[Av \wedge \exists w(Rwv \wedge Tasvw)]$
It could have been that everyone was taller than anyone is.	$\Diamond\forall x\otimes\forall y\otimes(Txy)$	$\exists v[Av \wedge \exists w(Rwv\wedge \forall x(Ixw \supset \forall y(Iyv \supset Txywv)))]$
It could have been that everyone who isn't a farmer was a farmer.	$\Diamond \otimes \forall x \otimes (\neg{\downarrow}Fx \supset {\uparrow}Fx)$	$\exists v[Av \wedge \exists w(Rwv\wedge \forall x(Ixw \supset (\neg Fxvv \supset Fxww)))]$

Table 1: Example Translations

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